Math Learning Disabilities

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While children with disorders in mathematics are specifically included under the definition of Learning Disabilities (Federal Register, August 23, 1977), seldom do math learning difficulties cause children to be referred for evaluation. In many school systems, special education services are provided almost exclusively on the basis of children's reading disabilities (Badian, 1983). Even after being identified as learning disabled (LD), few children are provided substantive assessment and remediation of their arithmetic difficulties (Goodstein & Kahn, 1974).

This relative neglect might lead parents and teachers to believe that arithmetic learning problems are not very common, or perhaps not very serious. However, approximately 6% of school-age children have significant math deficits (Kosc, 1974; Badian, 1983) and among students classified as learning disabled, arithmetic difficulties are as pervasive as reading problems (Badian, 1983; McKinney & Feagans, 1980). This does not mean that all reading disabilities are accompanied by arithmetic learning problems, but it does mean that math deficits are widespread and in need of equivalent attention and concern.

Evidence from learning disabled adults belies the social myth that it is okay to be rotten at math (Johnson & Blalock, 1987). The effects of math failure throughout years of schooling, coupled with math illiteracy in adult life, can seriously handicap both daily living and vocational prospects. In today's world, mathematical knowledge, reasoning, and skills are no less important than reading ability (Paulos, 1989; Steen, 1987; Stevenson, 1987).

DIFFERENT TYPES OF MATH LEARNING PROBLEMS

As with students' reading disabilities, when math difficulties are present, they range from mild to severe. There is also evidence that children manifest different types of disabilities in math (Badian, 1983; Cohn, 1971; Kosc 1974; Strang and Rourke, 1985). Unfortunately, research attempting to classify these has yet to be validated or widely accepted, so caution is required when considering descriptions of differing degrees of math disability. Still, it seems evident that students do experience not only differing intensities of math dilemmas, but also different types, which require diverse classroom emphases, adaptations and sometimes even divergent methods.

Mastering Basic Number Facts

Many learning disabled students have persistent trouble "memorizing" basic number facts in all four operations (Fleischner, Garnett, & Shepherd, 1982), despite adequate understanding and great effort expended trying to do so. Instead of readily knowing that 5+7=12, or that 4x6=24, these children continue laboriously over years to count fingers, pencil marks or scribbled circles and seem unable to develop efficient memory strategies on their own.

For some, this represents their only notable math learning difficulty and, in such cases, it is crucial not to hold them back "until they know their facts." Rather, they should be allowed to use a pocket-size facts chart in order to proceed to more complex computation, applications, and problem-solving. As the students demonstrate speed and reliability in knowing a number fact, it can be removed from a personal chart. Addition and multiplication charts also can be used for subtraction and division respectively. For specific use as a basic fact reference, a portable chart (back-pocket-size, for older students) is preferable to an electronic calculator. Having the full set of answers in view is valuable, as is finding the same answer in the same location each time since where something is can help in recalling what it is. Also, by blackening over each fact that has been mastered, overreliance on the chart is discouraged and motivation to learn another one is increased. For those students who have difficulty locating answers at the vertical/horizontal intersections, it helps to use cutout cardboard in a backward L-shape.

Several curriculum materials offer specific methods to help teach mastering of basic arithmetic facts (Garnett, Frank, & Fleischner, 1983; Thornton, 1978; Stern, 1987). The important assumption behind these materials is that the concepts of quantities and operations are already firmly established in the student's understanding. This means that the student can readily show and explain what a problem means using objects, pencil marks, etc. Suggestions from these teaching approaches include:.

Interactive and intensive practice with motivational materials such as games

... attentiveness during practice is as crucial as time spent

Distributed practice, meaning much practice in small doses

... for example, two 15-minute sessions per day, rather than an hour session every other day

Small numbers of facts per group to be mastered at one time

...and then, frequent practice with mixed groups-

Emphasis is on "reverses," or "turnarounds" (e.g., 4 + 5/5 + 4, 6x7/7x6)

...In vertical. horizontal, and oral formats

Student self-charting of progress

... having students keep track of how many and which facts are mastered and how many more there are to go

Instruction, not just practice

... Teaching thinking strategies from one fact to another (e.g., doubles facts, 5 + 5, 6 + 6, etc. and then double-plus-one facts, 5 + 6, 6 + 7, etc.).

(For details of these thinking strategies, see Garnett, Frank & Fleischner, 1983, Thornton.1978; or Stern, 1987).

Arithmetic Weakness/Math Talent

Some learning disabled students have an excellent grasp of math concepts, but are inconsistent in calculating. They are reliably unreliable at paying attention to the operational sign, at borrowing or carrying appropriately, and at sequencing the steps in complex operations. These same students also may experience difficulty mastering basic number facts.

Interestingly, some of the students with these difficulties may be remedial math students during the elementary years when computational accuracy is heavily stressed, but can go on to join honors classes in higher math where their conceptual prowess is called for. Clearly, these students should not be tracked into low level secondary math classes where they will only continue to demonstrate these careless errors and inconsistent computational skills while being denied access to higher-level math of which they are capable. Because there is much more to mathematics than right-answer reliable calculating, it is important to access the broad scope of math abilities and not judge intelligence or understanding by observing only weak lower level skills. Often a delicate balance must be struck in working with learning disabled math students which include:

(a) acknowledging their computational weaknesses

(b) maintaining persistent effort at strengthening inconsistent skills;

(c) sharing a partnership with the student to develop self-monitoring systems and ingenious compensations; and at the same time, providing the full, enriched scope of math teaching.

The Written Symbol System and Concrete Materials

Many younger children who have difficulty with elementary math actually bring to school a strong foundation of informal math understanding. They encounter trouble in connecting this knowledge base to the more formal procedures, language, and symbolic notation system of school math (Allardice & Ginsburg, 1983). The collision of their informal skills with school math is like a tuneful, rhythmic child experiencing written music as something different from what he/she already can do. In fact, it is quite a complex feat to map the new world of written -math symbols onto the known world of quantities, actions and, at the same time to learn the peculiar language we use to talk about arithmetic. Students need many repeated experiences and many varieties of concrete materials to make these connections strong and stable. Teachers often compound difficulties at this stage of learning by asking students to match pictured groups with number sentences before they have had sufficient experience relating varieties of physical representations with the various ways we string together math symbols, and the different ways we refer to these things in words. The fact that concrete materials can be moved, held, and physically grouped and separated makes them much more vivid teaching tools than pictorial representations. Because pictures are semiabstract symbols, if introduced too early, they easily confuse the delicate connections being formed between existing concepts, the new language of math, and the formal world of written number problems.

In this same regard, it is important to remember that structured concrete materials are beneficial at the concept development stage for math topics at all grade levels (Herbert, 1985; Suydam, 1984). There is research evidence that students who use concrete materials actually develop more precise and more comprehensive mental representations, often show more motivation and on-task behavior, may better understand mathematical ideas, and may better apply these to life situations (Harrison & Harrison, 1986; Suydam & Higgins, 1977). Structured, concrete materials have been profitably used to develop concepts and to clarify early number relations, place value, computation, fractions, decimals, measurement, geometry, money, percentage, number bases story problems, probability and statistics (Bruni & Silverman, 1986), and even algebra (Williams, 1986).

Of course, different kinds of concrete materials are suited to different teaching purposes (see appendix for selected listing of materials and distributors). Materials do not teach by themselves; they work together with teacher guidance and student interactions, as well as with repeated demonstrations and explanations by both teachers and students.

Often students' confusion about the conventions of written math notation are sustained by the practice of using workbooks and ditto pages filled with problems to be solved. In these formats, students learn to act as problem answerers rather than demonstrators of math ideas. Students who show particular difficulty ordering math symbols in the conventional vertical, horizontal, and multistep algorithms need much experience translating from one form to another. For example, teachers can provide answered addition problems with a double box next to each for translating these into the two related subtraction problems. Teachers can also dictate problems (with or without answers) for students to translate into pictorial form, then vertical notation, then horizontal notation. It can be helpful to structure pages with boxes for each of these different forms.

Students also can work in pairs translating answered problems into two or more different ways to read them (e.g., 20 x 56 - 1120 can be read twenty times fifty- six equals one thousand, one hundred and twenty or twenty multiplied by fifty-six is one thousand, one hundred, twenty). Or, again in pairs, students can be provided with answered problems each on an individual card; they alternate in their demonstration, or proof, of each example using materials (e.g., bundled sticks for carrying problems). To add zest, some of the problems can be answered incorrectly and a goal can be to find the "bad eggs."

Each of these suggestions is intended to move youngsters out of the rut of thinking of math as getting right answers or giving up. They help create a frame of mind that connects understanding with symbolic representation, while attaching the appropriate language variations.

The Language of Math

Some LD students are particularly hampered by the language aspects of math, resulting in confusion about terminology, difficulty following verbal explanations, and/or weak verbal skills for monitoring the steps of complex calculations. Teachers can help by slowing down the pace of their delivery, maintaining normal timing of phrases, and giving information in discrete segments. Such slowed down "chunking" of verbal information is important when asking questions, giving directions, presenting concepts, and offering explanations.

Equally important is frequently asking students to verbalize what they are doing (Lovitt & Curtiss, 1968). Too often, math time is filled either with teacher explanation or with silent written practice. Students with language confusions need to demonstrate with concrete materials and explain what they are doing at all ages and all levels of math work, not just in the earliest grades (Herbert, 1985). Having students regularly "play teacher" can be not only enjoyable but also necessary for learning the complexities of the language of math. Also, understanding for all children tends to be more complete when they are required to explain, elaborate, or defend their position to others; the burden of having to explain often acts as the extra push needed to connect and integrate their knowledge in crucial ways (Brown & Campione, 1986).

Typically, children with language deficits react to math problems on the page as signals to do something, rather than as meaningful sentences that need to be read for understanding. It is almost as though they specifically avoid verbalizing. Both younger and older students need to develop the habit of reading or saying problems before and/or after computing them. By attending to the simple steps of self-verbalizing, they can monitor more of their attentional slips and careless errors. Therefore, teachers should encourage these students to:

- Stop after each answer,
- Read aloud the problem and the answer, and
- Listen to myself and ask, "Does that make sense?"

For youngsters with language weakness, this may take repeated teacher modeling, patient reminding and much practice using a cue card as a visual reminder.

Visual-Spatial Aspects of Math

A small number of LD students have disturbances in visual-spatial-motor organization, which may result in weak or lacking understanding of concepts, very poor "number sense," specific difficulty with pictorial representations and/or poorly controlled handwriting and confused arrangements of numerals and signs on the page. Students with profoundly impaired conceptual understanding often have substantial perceptual-motor deficits and are presumed to have right hemisphere dysfunction (Strang & Rourke, 1985).

This small subgroup may well require a very heavy emphasis on precise and clear verbal descriptions. They seem to benefit from substituting verbal constructions for the intuitive/spatial/relational understanding they lack. Pictorial examples or diagrammatic explanations can thoroughly confuse them, so these should not be used when trying to teach or clarify concepts. In fact, this subgroup is specifically in need of remediation in the area of picture interpretation, diagram and graph reading, and nonverbal social cues (Johnson, 1987). To develop an understanding of math concepts, it may be useful to make repeated use of concrete teaching materials (e.g., Stern blocks, Cuisenaire rods), with conscientious attention to developing stable verbal renditions of each quantity (e.g., 5), relationship (e.g., 5 is less than 7), and action (e.g., 5+2=7). Since understanding visual relationships and organization is difficult for these students, it is important to anchor verbal constructions in repeated experiences with structured materials that can be felt, seen, and moved around as they are talked about. For example, they may be better able to learn to identify triangles by holding a triangular block and saying to themselves, "A triangle has three sides. When we draw it, it has three connected lines." For example, a college freshman who had this deficit could not "see" what a triangle was without saying this to herself when she looked at different figures or attempted to draw a triangle.

The goal for these students is to construct a strong verbal model for quantities and their relationships in place of the visual-spatial mental representation that most people develop. Consistent descriptive verbalizations also need to become firmly established in regard to when to apply math procedures and how to carry out the steps of written computation. Great patience and verbal repetition are required to make small incremental steps.

It is important to recognize that average, bright, and even very bright youngsters can have the severe visual-spatial organization deficits that make developing simple math concepts extremely difficult. When such deficits are accompanied by strong verbal skills, there is a tendency to disbelieve the impaired area of functioning. Thus, parents and teachers can spend years growling, "She's just not trying ... She doesn't play attention . . . She must have a math phobia. . . It's probably an emotional problem." Because other accompanying weaknesses usually include a poor sense of body in space, difficulty reading the nonverbal social signals of gesture and face, and often nightmarish disorganization in the world of "things," it can be easy to mistake the problem for a constellation of emotional symptoms. Misreading the problems in this way delays the appropriate work that is needed both in mathematics and the other areas.

IN SUMMARY

Math learning difficulties are common, significant, and worthy of serious instructional attention in both regular and special education classes. Students may respond to repeated failure with withdrawal of effort, lowered self-esteem, and avoidance behaviors. In addition, significant math deficits can have serious consequences on the management of everyday life as well as on job prospects and promotion.

Math learning problems range from mild to severe and manifest themselves in a variety of ways. Most common are difficulties with efficient recall of basic arithmetic facts and reliability in written computation. When these problems are accompanied by a strong conceptual grasp of mathematical and spatial relations, it is important not to bog the student down by focusing only on remediating computation. While important to work on, such efforts should not deny a full math education to otherwise capable students.

Language disabilities, even subtle ones, can interfere with math learning. In particular, many LD students have a tendency to avoid verbalizing in math activities, a tendency often exacerbated by the way math is typically taught in America.

Developing their habits of verbalizing math examples and procedures can greatly help in removing obstacles to success in mainstream math settings.

Many children experience difficulty bridging informal math knowledge to formal school math. To build these connections takes time, experiences, and carefully guided instruction. The use of structured, concrete materials is important to securing these links, not only in the early elementary grades, but also during concept development stages of higher-level math. Some students need particular emphasis on the translating between different written forms, different ways of reading these, and various representations (with objects or drawings) of what they mean.

An extremely handicapping, though less common math disability, derives from significant visual-spatial-motor disorganization. The formation of foundation math concepts is impaired in this small subgroup of students. Methods to compensate include avoiding the use of pictures or graphics for conveying concepts, constructing verbal versions of math ideas, and using concrete materials as anchors. The organizational and social problems that accompany this math disability are also in need of long-term appropriate remedial attention in order to support successful life adjustment in adulthood.

In sum, as special educators, there is much we can and need to do in this area that calls for so much greater attention than we have typically provided.

References

Allardice, B.S., & Ginsburg, H.P. (1983). Children's psychological difficulties in mathematics. In Ginsburg, H.P. (Ed.) The development of mathematical thinking. New York: Academic Press.

Badian N.A. (1983). Arithmetic and nonverbal learning. In Myklebust, H.R., (Ed.), Progress in Learning Disabilities, Volume V. New York: Grunt & Stratton.

Brown, A.L., & Campione, J.C. (1986). Psychological theory and the study of learning disabilities. American Psychologist, 14, 1059-1068.

Bruni, J.V., & Silverman, H.J. (1986). Developing concepts in probability and statistics - and much more. Arithmetic Teacher 33,34-37.

Cohn, R. (1971). Arithmetic and learning disabilities. In Myklebust, M.R., (Ed.) Progress In learning disabilities, VOL 11, New York: Grunt & Stratton.

DFleischner, J.E., Garnett, K., & Shepherd, M.J. (1982). Proficiency in basic fact computation of learning disabled and non-disabled children. Focus on Learning Problems In Mathematics 4,47-55.

Garnett, K., Frank, B., & Fleischner, JX (1983). A strategies generalization approach to basic fact learning (addition and subtraction lessons, manual #3 multiplication lessons, manual #5). Research Institute for the Study of Learning Disabilities. New York: Teachers College, Columbia University.

Goodstein, M.A., & Kahn, H. (1974). Pattern of achievement among children with learning difficulties. Exceptional Children. 50.47-49.

Harrison, M., & Harrison, B. (1986). Developing numeration concepts and skills. Arithmetic Teacher, 33,18-21, SO.

Herbert, E. (1985). One point of view: Manipulatives are good mathematics. Arithmetic Teacher, 32.4.

Johnson, .0. (1987). Nonverbal learning disabilities. Pediatric Annuals, 18, 133-141.

Johnson, D., & Blalock, J. (1967). Adults with learning disabilities: clinical studies. Orlando, FL Grunt & Stratton

Kosc, L. (1974). Developmental dyscalculia. Journal of Learning Disabilities, 7,164-177.

Lovitt, T.C., & Curtiss, K.A. (1968). Effects of manipulating an antecedent event on mathematics response rate. Journal of Applied Behavior Analysis, 1, 329-333.

McKinney, J.D., & Feagans, L. 1980 Learning Disabilities In the Classroom (Final project report). Chapel Hill: University of North Carolina, Frank Porter Graham Child. Development Center.

Paulos, J.A. , The odds are you're Innumerate. Now York Times, Book Review, January 1, 1989.

Steen, LA.. (1987). Mathematics education: A predictor of scientific competitiveness. Science, 237,251-252,302.

Stem, M. (1967). Experimenting with numbers (rev. ed.) Cambridge MA: Educators Publishing Service.

Strang, JD, & Rourke, B.P. (1985). Adaptive behavior of children who exhibit specific arithmetic disabilities and associated neuropsychological abilities and deficits in BP Rourke (Et) Neuropsychology of learning disabilities. New York: The Guilford Press

Suydam, M.N. (1984). Research Report: Manipulative Materials. Arithmetic Teacher, 31, 27.

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